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Energy efficiency in cognitive radio network: study of cooperative sensing using different channel sensing methods

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Energy efficiency in cognitive radio network: Study of cooperative sensing using different channel sensing methods

by

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Ahmed El-Sayed Kamal
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DEDICATION

I dedicate my thesis work to my loving family, especially to my mom Ke Chen and my dad Jianchun Cui, who have been loving and supporting me since the beginning of my life.

I dedicate my thesis work to all of my friends, we together have created our best memories and we have taken care of each other in hard times.

I also dedicate my thesis work to my professor Sang Kim, who has given me knowledge and always been patient with me in my difficult time.

Last but not least, I dedicate my thesis work to you, because somehow we are all connected.

TABLE OF CONTENTS

NOMENCLATURE AND NOTATION

ADC	Analog to Digital Converter
CR	Cognitive Radio
FC	Data Fusion Center
PU	Primary Users
SINR	Signal to Interference plus Noise Ratio
SNR	Signal to Noise Ratio
SR	Secondary Receiver
SU	Secondary User
H_0	Hypothesis Test 0
H_1	Hypothesis Test 1
$s(t)$	Primary Signal
$n(t)$	Noise Signal
σ_n^2	Noise Variance
T_s	Sensing Time
B	Sensing Bandwidth
λ	Detection threshold
γ	SNR from PU and Measured at CR
P_{CS}^h	Signal Power Transmitted from CR and Measured at SR
P_P^h	Signal Power Transmitted from PU and Measured at CR
C_{ave}	Average CR Channel Capacity
E_{total}	CR Network Total Energy Consumption

P_T	CR Transmission Power
P_S	CR Sensing Power
T_T	CR Tranmission Time
T_S	CR Sensing Time
E_d	CR Decision Report Signal Energy Consumption
E_C	CR Circuit Energy Consumption
K	Number of Cooperative Sensing CR
D_{FC}	Sum of Decision Bits from CR to DFC
P_e	Average Reporting Channel Error Probability
P_f	False Alarm Probability at Each CR
n	Decision Threshold
S	Sensed Channels
k	Number of PU That Are Active (Transmitting)
P_m	Misdetection Probability
$P_{m,r,k}$	Misdetection Probability for r PUs When k of Them Are Active
$P_{m,r}$	Total Misdetection Probability for r PUs at CR
Q_f	Network False Alarm Probability
Q_m	Network Misdetection Probability
$Q_{f,r}$	Network False Alarm Probability for r PUs
$Q_{m,r,k}$	Network Misdetection Probability When k out of r PUs Are Active
$\in_m$	Network Misdetection Threshold
$\in_{m,CR}$	Misdetection Threshold on Each CR

ABSTRACT

When cognitive radio (CR) operates, it starts by sensing spectrum and looking for idle bandwidth. There are several methods for CR to make a decision on either the channel is occupied or idle, for example, energy detection scheme, cyclostationary detection scheme and matching filtering detection scheme [1]. Among them, the most common method is energy detection scheme because of its algorithm and implementation simplicities [2]. There are two major methods for sensing, the first one is to sense single channel slot with varying bandwidth, whereas the second one is to sense multiple channels and each with same bandwidth. After sensing periods, samples are compared with a preset detection threshold and a decision is made on either the primary user (PU) is transmitting or not. Sometimes the sensing and decision results can be erroneous, for example, false alarm error and misdetection error may occur. In order to better control error probabilities and improve CR network performance (i.e. energy efficiency), we introduce cooperative sensing; in which several CR within a certain range detect and make decisions on channel availability together. The decisions are transmitted to and analyzed by a data fusion center (DFC) to make a final decision on channel availability. After the final decision is been made, DFC sends back the decision to the CRs in order to tell them to stay idle or start to transmit data to secondary receiver (SR) within a preset transmission time. After the transmission, a new cycle starts again with sensing.

In this thesis, we find methods to maximize total energy efficiency of the cognitive radio network using closed form expressions, and with the consideration on misdetection threshold on each CR or the CR network in order to protect PU. Furthermore, we derives the

optimal fusion rule for DFC in order to maximize energy efficiency in a cooperative sensing environment. Finally compare between two difference sensing schemes and find the one which further maximize the energy efficiency.

This thesis report is organized as followed: Chapter II review some of the papers on optimizing CR energy efficiency. In Chapter III, we study how to achieve maximal energy efficiency when CR senses single channel with changing bandwidth and with constrain on misdetection threshold in order to protect PU; furthermore, a case study is given and we calculate the energy efficiency. In Chapter IV, we study how to achieve maximal energy efficiency when CR senses multiple channels and each channel with same bandwidth, also, we preset a misdetection threshold and calculate the energy efficiency. A comparison will be shown between two sensing methods at the end of the chapter. Finally, Chapter V concludes this thesis.

CHAPTER I

INTRODUCTION

The Federal Communication Commission has allocated radio spectrum from 9 KHz to 275 GHz [3], and with increasing amount of wireless transceiver devices and networks, we will encounter difficulties in assigning and licensing new wireless users due to scarcity in available spectrum in the near future. On the other hand, according to a survey done in Dublin Ireland [14], more than 80 percent of spectrum is underutilized, and similarly for some major cities in the United States. Therefore, there is a low spectrum utilization rate.

Joseph Mitola III, from Stevens Institute of Technology, foresaw this problem and developed the concept of cognitive radio in the late 1990s and this technology has since attracted much attentions among scholars all around the world. CR does not have a license for any radio frequency band. It accesses to channels opportunistically when the licensed user (PU) is not transmitting its signal. And jumps out from that channel in 2 seconds (IEEE 802.22) after PU restarts to transmit its signal. A similar scenario is that imagine you are at a movie theatre on your own and you forget to purchase a ticket. You want to sit down to watch the movie but the theatre is full. One way to watch the movie while sitting down is to wait by the entrance door until someone stands up to go to use the restroom or answer an important phone call. Then you can run to his or her seat and sit for a while until he or she comes back, and you have to leave the seat and wait by the entrance for the next opportunity. Besides alleviating spectrum scarcity issue, CR can be used for secured communication, since CR search for available spectrum on itself and it is a type of frequency hopping radio, we can apply this technology for secured communication such as on the battlefields.

CHAPTER II

LITERATURE REVIEW ON CR ENERGY EFFICIENCY

In this chapter, we choose and review some of the papers [4] [5] [6] [7] on optimizing energy efficiency in cognitive radio network. Some papers are on optimizing energy efficiency in sensing period [4] [5], some on energy efficiency optimization in transmission period [6], and finally a paper on CR overall circuit energy consumption by studying the tradeoff between sensing and transmission of the cognitive radio [7].

Paper [4] proposed a method on optimizing the energy efficiency (or opportunity cost, as described in the paper) in sensing period of the cognitive radio network. In the paper, sensing period is divided into R rounds, and at each round, CR is able to make a local decision on either the PU is transmitting or in idle for a specific channel. Each round is consisted of three parts, spectrum sensing, decision report, and channel switching. Therefore, CR is able to sense R channels during sensing period. After all CRs report their decision to DFC, the DFC use a specific data fusion rule to produce a final decision on either PU is transmitting or not. The sensing energy efficiency is described using total energy consumption (sum of energy consumption in spectrum sensing, decision report and channel switching) divides the amount of available bandwidth found in R rounds. The metrics of energy efficiency is in Joule per Hz. The paper later optimized such energy efficiency with the consideration of preset misdetection and false alarm probability.

Another paper to mention on sensing energy efficiency is [5]. The authors proposed an energy efficiency sensing scheme by using adaptive spectrum sensing algorithm and sequential sensing policy [5]. The adaptive spectrum sensing algorithm was introduced since PU's active and idle probability and periods can be studied [5], in order to allow CR to limit

sensing time and give more opportunity for transmission and reduce missed spectrum opportunities. On the other hand, sequential sensing policy is to constrain two error probability, namely false alarm probability and misdetection probability. The authors set two threshold for detection, λ_1 and λ_2. If received signal energy is greater than λ_2, then CR decides that PU is transmitting, whereas if received signal energy is less than λ_1, CR decides PU is not transmitting. And finally if received signal energy is between two thresholds, CR will collect more signal samples. By optimizing these two threshold, the paper shows that we can minimizing false alarm probability, which allow more transmission opportunities for CR, and minimizing misdetection probability below a certain threshold in order to protect PU.

Paper [6] introduced a method to optimize transmission energy efficiency by optimizing transmission duration and power allocation to each channel. First, the authors showed the model for energy efficiency for CR in a frame cycle, and it was a function of transmission duration. Then they derived the derivative of such energy efficiency with respect to transmission time and set the function equals to zero to find the optimal transmission duration. Next, the authors moved to the other topic, which is to find the optimal power allocation for K channels under a preset transmission power budget. By using gradient assisted binary search and binary search assisted ascent [8], the authors showed that there exist an optimal power allocation in transmission period for cognitive radio.

Last but not least, Paper [7] maximize the overall energy efficiency of cognitive radio (both in sensing and transmission periods). The authors assumed power consumption during sensing period is mainly caused by low noise amplifier and analog to digital converter. Power amplifier, low noise amplifier and analog to digital converter contributes to the power consumption in transmission periods. Later on, the authors showed that in order to maximize

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energy efficiency, one should maximize sensing time, the ratio of the power consumption for power amplifier and analog to digital converter, bit resolution of the ADC and the input backoff of the power amplifier [7]. Since there is no close form expression for their energy efficiency expression. The authors used numerical simulation to find the maximal energy efficiency.

In general, this literature review shows that on the topic of energy efficiency maximization for cognitive radio network, some research has been done on energy efficiency maximization in sensing period, transmission period and overall energy efficiency with the consideration in misdetection probability but without close form expression for maximal energy efficiency. In this thesis work, we analyze two different sensing methods for CR at each sensing period: sensing single channel with changing bandwidth and sensing multiple channels each with same bandwidth. After this, we find the optimal fusion rule with constrain on misdetection probability using misdetection threshold for each sensing case. And finally, we simulate the performance of energy efficiency. Therefore, compare with other works on energy efficiency in cognitive radio network, which only maximize sensing or transmission energy efficiency, or maximize overall energy efficiency but without a close form expression; the contribution of this work is to find methods to maximize total energy efficiency of the cognitive radio network using closed form expressions with a misdetection threshold on each CR or the CR network in order to protect PU. Furthermore, we derives the optimal fusion rule for DFC which helps CR network to maximize its energy efficiency, and finally we compare between two difference sensing schemes and analyze their performance on maximizing energy efficiency.

CHAPTER III

SENSING SINGLE CHANNEL OF VARYING BANDWIDTH

Energy Detection Scheme and Error Probabilities

In this section, we first derive the pdf and CDF of received input samples. Assume hypothesis test 1 (H_1) is when PU is transmitting its signal, hypothesis test 0 (H_0) is when PU is not transmitting its signal. Assume CR receives inputs without PU signal (H_0) are Gaussian random process with zero mean, $x(t) = n(t)$: $\mathcal{N} \sim (0, \sigma_n^2)$. And Assume CR received inputs with PU signal (H_1) present is Gaussian random process with nonzero mean $x(t) = \alpha \cdot s(t) + n(t)$: $\mathcal{N} \sim (\mu_s, \sigma_n^2)$. With some channel fading factor α.

$$x(t) = \begin{cases} \alpha \cdot s(t) + n(t) & H_1 \\ n(t) & H_0 \end{cases} \tag{1.1}$$

According to Claude Shannon's sampling theorem [9], for a signal with bandwidth B and is sampled in a time T_s, we need $2T_sB$ samples in order to acquire knowledge of the received signal completely. The sampled signals are:

$$x(t)' = \begin{cases} \displaystyle\sum_{i=1}^{2T_sB} (\alpha \cdot s_i(t) + n_i(t)) & H_1 \\ \displaystyle\sum_{i=1}^{2T_sB} n_i(t) & H_0 \end{cases} \tag{1.2}$$

Assume noise is additive white Gaussian noise with zero mean and variance σ_n^2, and each sample is a random variable with zero mean and variance $\sigma_{i,n}^2 = \sigma_n^2$, we divide every samples by noise standard deviation σ_n; then the normalized noise sample has distribution of $\mathcal{N} \sim (0,1)$. For energy detection, we square and sum each received signal sample with unit variance, and result a sum of the squares of $2T_sB$ i.i.d random variables:

$$y(t) = \begin{cases} \sum_{i=1}^{2T_sB} \left(\frac{\alpha \cdot s_i(t) + n_i(t)}{\sigma_n} \right)^2 & H_1 \\ \sum_{i=1}^{2T_sB} \left(\frac{n_i(t)}{\sigma_n} \right)^2 & H_0 \end{cases} \tag{1.3}$$

For H_0, we have squares of $2T_sB$ standard normalized Gaussian random variables, each with zero mean and unit variance. Therefore, it follows central chi-squared distribution with $2T_sB$ degrees of freedom. The pdf of such distribution according to reference [10] is:

$$f_Y(y, H_0) = \frac{1}{2^{T_sB}\Gamma(T_SB)} y^{T_sB-1} e^{-\frac{y}{2}} \tag{1.4}$$

The complimentary CDF of the distribution at point λ is the false alarm probability [10]:

$$P_f = 1 - F_Y(\lambda, H_0) = \frac{\Gamma\left(T_sB, \frac{\lambda}{2}\right)}{\Gamma(T_sB)} \tag{1.5}$$

In which $\Gamma(.)$ is gamma function and $\Gamma(., \mathrm{b})$ is upper incomplete gamma function at point b. In the case of PU transmitting, Assume primary signal and noise are independent. Sampled normalized signals have means of $\mu_s = \mu_{s,i} = \frac{1}{2T_sB} \sum_{1}^{2T_sB} \left(\frac{\alpha \cdot s_i(t)}{\sigma_n} \right)$ and variances of 0. Then the received inputs are normal distributed $\mathcal{N} \sim (\mu_s, 1)$. Which follows non-central chi-squared distribution with $2T_sB$ degrees of freedom. And the pdf of such distribution is:

$$f_Y(y, H_1) = \frac{1}{2} e^{-\frac{y+\varphi}{2}} \left(\frac{y}{\varphi} \right)^{\frac{T_sB-1}{2}} I_{T_sB-1}\left(\sqrt{\varphi y} \right) \tag{1.6}$$

In which φ is the non-centrality parameter and $I_{a-1}\left(\sqrt{b \cdot c} \right)$ is the modified Bessel's function.

We have the misdetection probability (P_m) of each CR according to reference [10]:

$$P_m = 1 - Q_{T_sB}\left(\sqrt{2\gamma}, \sqrt{\lambda} \right) \tag{1.7}$$

In which $Q_a(\sqrt{b}, \sqrt{c})$ is Marcum-Q function and γ is SNR from PU and measured at CR. For now, we have derived pdf and CDF of received input samples. In detection theory,

we always measure the performance of detector using false alarm probability (probability of CR decides PU is transmitting, given PU is actually not transmitting: $P_f = P(\widehat{H_1}|H_0)$; and misdetection probability (probability of CR decides PU is not transmitting, given PU is actually transmitting: $P_m = P(\widehat{H_0}|H_1)$). Accordingly, given a preset detection threshold λ for $2T_sB$ samples, which CR decides PU is transmitting if the sum of received signal power is above the threshold, and decides PU is idle if the sum is below λ, we can easily find out P_f and P_m for each CR using CDF derived in equations (1.5) and (1.7).

$$\begin{cases} P_f = \text{P}(y > \lambda|H_0) = 1 - F_Y(\lambda, H_0) = \dfrac{\Gamma(T_sB, \frac{\lambda}{2})}{\Gamma(T_sB)} \\ P_m = \text{P}(y < \lambda|H_1) = 1 - Q_{T_sB}\left(\sqrt{2\gamma}, \sqrt{\lambda}\right) \end{cases} \tag{1.8}$$

Network Channel Capacity and Energy Efficiency

In this section, we want to finally derive energy efficiency (bits per Joule) of the CR network. But first, we calculate transmission capacity of CR under correct detection $(H_0, \widehat{H_0})$ and misdetection $(H_1, \widehat{H_0})$; given sensing bandwidth W, detection threshold λ, amount of CRs and other parameters.

Assume the signal power transmitted to SR from CR, and measured at SR is P_{CS}^h with channel gain h. Assume sensing bandwidth is B and noise spectrum power is N_0. CR transmission channel capacity of correct detection is:

$$C_d = Blog_2\left(1 + \frac{P_{CS}^h}{N_0B}\right)$$

Given the probability of CR transmission under this condition is $P(H_0, \widehat{H_0})$ $= P(\widehat{H_0}|H_0)P(H_0)$, the weighted transmission capacity of correct detection is therefore:

$$C_{d(weighted)} = B log_2\left(1 + \frac{P_{CS}^h}{N_0 B}\right) P(\widehat{H_0}|H_0)P(H_0) \qquad (1.9)$$

Similarly, the transmission capacity of misdetection is:

$$C_{md} = B log_2\left(1 + \frac{P_{CS}^h}{N_0 B + P_P^h}\right)$$

In which P_P^h is signal power from PU measured at secondary network (CR-SU network) and g is the channel gain from PU to secondary network. Probability of transmission under misdetection is $P(H_1, \widehat{H_0}) = P(\widehat{H_0}|H_1)P(H_1)$. The weighted transmission capacity of misdetection is:

$$C_{md(weighted)} = B log_2\left(1 + \frac{P_{CS}^h}{N_0 B + P_P^h}\right) P(\widehat{H_0}|H_1)P(H_1) \qquad (1.10)$$

Assume in cooperative sensing case, Q_f is network false alarm probability and Q_m is network misdetection probability, we have $P(\widehat{H_0}|H_0) = (1 - Q_f)$, and $P(\widehat{H_0}|H_1) = Q_m$, and the average CR channel capacity is:

$$C_{ave} = B\left[log_2\left(1 + \frac{P_{CS}^h}{N_0 B}\right)(1 - Q_f)P(H_0) + log_2\left(1 + \frac{P_{CS}^h}{N_0 B + P_P^h}\right)Q_m P(H_1)\right] \qquad (1.11)$$

Next, we give the energy consumption of CR network. The total energy consumption of the network in one frame cycle is:

$$E_{total} = P_T T_T + (P_S T_S + E_d + \mathrm{E_C})K \qquad (1.12)$$

Equation (1.12) shows the total energy consumption of cooperative sensing CR network, P_S is sensing power; T_S is sensing time; $P_T T_T$ is transmission energy from CR to SR. E_d is one bit decision report signal energy from every CR to DFC; and finally E_C is circuit energy consumption of CR. Hence; average energy efficiency in bits per watt is:

$$Average\ energy\ efficiency = \frac{T_T C_{ave}}{E_{total}} \qquad (1.13)$$

In the next section, we look at some famous data fusion rules such as AND fusion rule and OR fusion rule; then we show the fusion rule that can achieve maximal energy efficiency given preset misdetection threshold, in order to protect PU.

Cooperative Sensing and Data Fusion Rules

In the first section, we derived probability of false alarm and misdetection of each CR during sensing period. Next, we discuss about different data fusion rules and network false alarm and misdetection probability.

Assume there are K CRs nearby which are able to join cooperative sensing network (Fig.1), and each CR sends one bit decision K_i $(i = 1,2,\ldots,K;\ K_i = 0\ or\ 1)$ to DFC. The DFC receives Kk binary decisions and then make a final decision using these binary data. Assume decision threshold is n, and summation of received bits add up to be D_{FC}. If the summation of the received bits is less than decision threshold n, DFC decides PU is not transmitting. If the summation of the received bits is greater than or equals to decision threshold n, the DFC decides PU is transmitting. We have:

$$D_{FC} = \begin{cases} \sum_{i=1}^{K} K_i < n, & \mathcal{H}_0 \\ \sum_{i=1}^{K} K_i \geq n, & \mathcal{H}_1 \end{cases} \qquad (1.14)$$
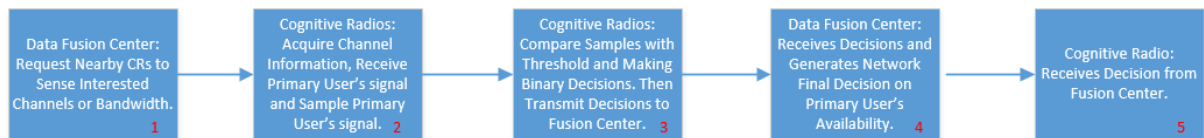
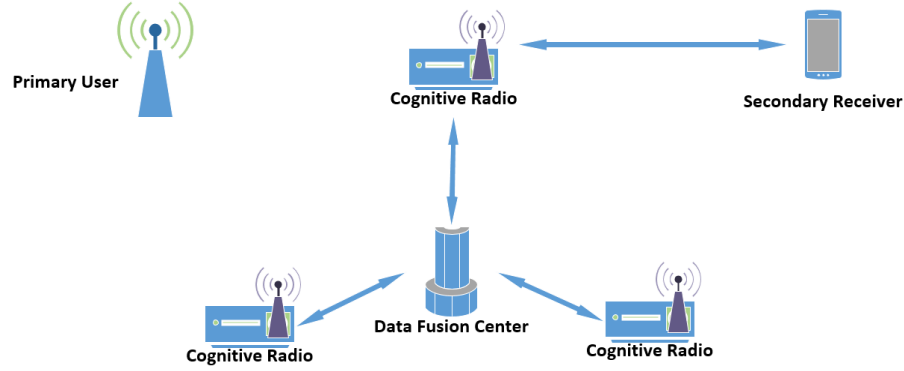

Figure 1.1 a: CR Cooperative Sensing Network Diagram

Figure 1.1 b: CR Cooperative Sensing Network Schematics

From previous studies on data fusion rules introduced in [10] and [11], we generalize following commonly used fusion rules:

'OR' Fusion Rule

The one bit data sent from each CR to DFC can be either 0 (PU idle) or 1 (PU transmitting). 'OR' fusion rule allows DFC to make final decisions of PU is transmitting if one or more CR out of Kk CRs decide(s) PU is transmitting [10]. Assume Q_f and Q_m are the false alarm and misdetection probability in DFC's final decisions; we have:

$$\begin{cases} Q_f = 1 - \prod_{i=1}^{K}(1 - P_{f,i}) = 1 - \prod_{i=1}^{K}\left(1 - \frac{\Gamma\left(T_s W, \frac{\lambda}{2}\right)}{\Gamma(T_s W)}\right) \\ Q_m = \prod_{i=1}^{K}(P_{m,i}) = \prod_{i=1}^{K}\left(Q_{T_s W}(\sqrt{2\gamma}, \sqrt{\lambda})\right) \end{cases} \tag{1.15}$$

Assume reporting channel from each CR to DFC is also fading channel with error probability $P_{e,i}$ (imperfect reporting channel), and reporting errors in all channels are identical and independent. Then we have $P_{e,i}$ (for i = 1,2, ..., K) = P_e(Average reporting channel error probability) and assume each CR has same P_f and P_m:

$$let \begin{cases} P_f^e = P_f(1 - P_e) + P_e(1 - P_f) \\ P_m^e = P_m(1 - P_e) + P_e(1 - P_m) \end{cases}$$

we have $\begin{cases} Q_f = 1 - (1 - P_f^e)^K \\ Q_m = (P_m^e)^K \end{cases}$

$\begin{cases} Q_f = 1 - [(1 - P_f) \cdot \\ Q_m = [P_m \cdot (1 - P_e) \cdot \end{cases}$

'AND' Fusion Rule

'AND' fusion rule allows DFC to make final decisions of PU is transmitting if and only if all of the CRs decide(s) PU is transmitting. If even one CR decides PU is idle, the DFC will make a final decision that PU is idle [10]. 'AND' fusion rule is described below:

$\begin{cases} Q_f = (P_f^e)^K \\ Q_m = 1 - (1 - P_m^e)^K \end{cases}$

$\begin{cases} Q_f = [P_f \cdot (1 - P_e) + P_e \cdot (1 - P_f)]^k \\ Q_m = 1 - [(1 - P_m) \cdot (1 - P_e) + P_e \cdot P_m]^k \end{cases}$

n Out of K Fusion Rule

When the amount of CRs that join cooperative sensing is K, and decision threshold is n, we have the general expression of the network false alarm and misdetection probability for n out of K fusion rule as described in equation (1.18):

$$\begin{cases} Q_f = \sum_{l=n}^{K} \binom{K}{l} (P_f^e)^l \cdot (1 - P_f^e)^{K-l} \\ Q_m = 1 - \sum_{l=n}^{K} \binom{K}{l} (1 - P_m^e)^l \cdot (P_m^e)^{K-l} \end{cases} \tag{1.18}$$

Which OR fusion rule is equivalent as letting $n = 1$ in equation (1.18), and AND fusion rule is same as letting $n = K$ in equation (1.18).

Study on AND Fusion Rule with Constrain on Misdetection Probability

Among above fusion rules, we want to find the one which gives maximal energy efficiency. According to equation (1.13), given the number of coop CR, we want to minimize false alarm probability while maximize misdetection probability in order to maximize energy efficiency. According to the general equations for Q_f and Q_m given in equation (1.18), we have:

$$\begin{cases} Q_{f,(n=K)} = \sum_{l=K}^{K} \binom{K}{l} (P_f^e)^l (1 - P_f^e)^{K-l} = (P_f^e)^K & for\ n = K \\ Q_{f,(n<K)} = (P_f^e)^K + \sum_{l=n}^{K-1} \binom{K}{l} (P_f^e)^l (1 - P_f^e)^{K-l} & for\ n < K \end{cases} \quad (1.19)$$

Clearly, $Q_{f,(n<K)}$ is always greater than $Q_{f,(n=K)}$, since the second term on the right side of equation for $Q_{f,(n<K)}$ is always greater than 0 (for nonzero false alarm probability at each CR). Therefore, for given K, in order to minimize false alarm probability, we should use 'AND' fusion rule, in which decision threshold is equal to number of coop CRs.

Next, we want to maximize misdetection probability:

$$\begin{cases} Q_{m,(n=K)} = 1 - (1 - P_m^e)^K & for\ n = K \\ Q_{m,(n<K)} = 1 - (1 - P_m^e)^K - \sum_{l=n}^{K-1} \binom{K}{l} (1 - P_m^e)^l (P_m^e)^{K-l} & for\ n < K \end{cases} \quad (1.20)$$

Clearly, $Q_{m,(n<K)}$ is always less than $Q_{m,(n=K)}$, since the third term on the right side of equation for $Q_{m,(n<K)}$ is always greater than 0 (for nonzero misdetection probability at each CR). In conclusion, for given K, in order to maximize misdetection probability, we should let $n = K$, which is equivalent as using 'AND' fusion rule.

Furthermore, Figure 1.2 below shows the decrease of false alarm probability and increase of misdetection probability as the number of CRs increase; by setting $P_m^e = P_f^e = 0.01$. In real applications, we want to transmit using maximal energy efficiency with preset threshold for misdetection probability $\in_m$, in order to protect PU. Next, we study how to achieve maximal energy efficiency using AND fusion rule given $\in_m$.
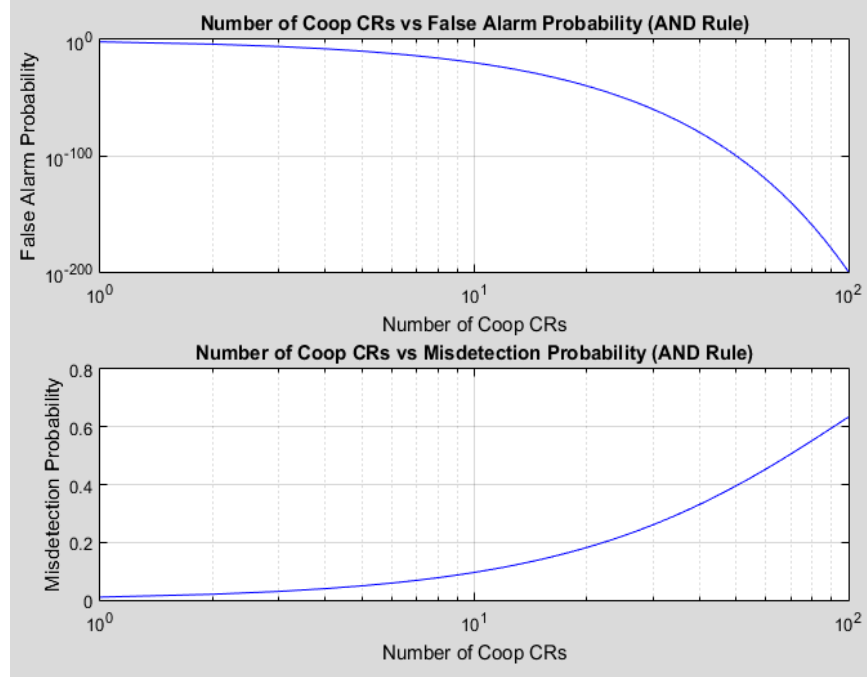
Figure 1.2: False Alarm Probability and Misdetection Probability

According to equation (1.17), in order to let the network misdetection probability less than or equal to $\in_m$, misdetection probability at each CR should satisfy:

$$\begin{cases} \in_m \geq Q_m = 1 - (1 - P_m^e)^K \\ P_m^e \leq 1 - (1 - \in_m)^{\left(\frac{1}{K}\right)} \end{cases} \tag{1.21}$$

From page 8 we have:

$$P_m = \frac{P_m^e - P_e}{1 - 2P_e} \leq \frac{1 - (1 - \in_m)^{\left(\frac{1}{K}\right)} - P_e}{1 - 2P_e} \tag{1.22}$$

And also from equation (1.7), we have

$$P_m = 1 - Q_{T_sB}\left(\sqrt{2\gamma}, \sqrt{\lambda}\right) = ncx2cdf(\lambda, T_SB, T_SB\gamma) \tag{1.23}$$

In which

$$ncx2cdf(a, b, c) = \sum_{j=0}^{\infty} \left(\frac{\left(\frac{1}{2}c\right)^j}{j!} e^{-\frac{c}{2}} \right) \Pr\left[x_{b+2j}^2 \leq x\right] \tag{1.24}$$

Equation (1.24) computes the non-central chi square cumulative distribution function at a, using degrees of freedom b and non-centrality parameter c [12]. And P_m increases as λ increases. Next, according to [13] we have:

$$\lambda = ncx2inv(P_m, \text{T}_S B, \text{T}_S B\gamma) \qquad (1.25)$$

Which $ncx2inv(P_m, \text{T}_S B, \text{T}_S B\gamma)$ is inverse non-central chi square cumulative distribution function. Notice that λ increases as P_m increases. Therefore; by substituting P_m with right hand side of equation (1.18) we have:

$$\lambda \leq ncx2inv\left(\frac{1 - (1 - \in_m)^{\left(\frac{1}{K}\right)} - P_e}{1 - 2P_e}, \text{T}_S B, \text{T}_S B\gamma\right) \qquad (1.26)$$

Furthermore, in equation (1.5), false alarm probability decreases as detection threshold λ increases (in which $1 - P_f$ increases as λ increases). Therefore, in AND fusion rule, in order to minimize false alarm probability and maximize misdetection probability at each CR while ensuring misdetection probability is below preset misdetection threshold $\in_m$, we should maximize detection threshold λ and the maximal value is:

$$\lambda_{max} = ncx2inv\left(\frac{1 - (1 - \in_m)^{\left(\frac{1}{K}\right)} - P_e}{1 - 2P_e}, \text{T}_S B, \text{T}_S B\gamma\right) \qquad (1.27)$$

For now, we have learned how to maximize energy efficiency, which is to use AND fusion rule, and derived the optimal detection threshold in order to maximize energy efficiency while ensuring misdetection probability is below misdetection threshold, in order to protect PUs. Next, we provide a case study and calculate the maximal energy efficiency.
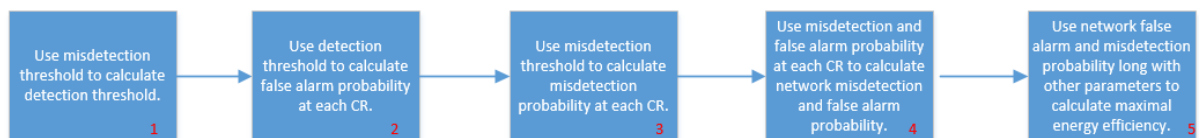

Figure 1.3: *Flow Chart on Steps to Calculate Energy Efficiency*

Case Study: Maximal Energy Efficiency Using AND Fusion Rule with
Misdetection Threshold

Setting-up the Parameters

Table 1.1: Spectrum Occupancy in Dublin, Ireland [14]

Start Freq (MHz)	Stop Freq (MHz)	Span (MHz)	Spectrum Band Allocation	Spectrum Fraction Used	Occupied Spectrum (MHz)	Average Percent Occupied
30	54	24	Fixed Mobile	0.056309	1.351416	5.6
54	88	34	Fixed Mobile	0.18486	6.28524	18.5
108	138	30	Aeronautical	0.062866	1.88598	6.3
138	174	36	Aeronautical	0.12546	4.51656	12.5
174	216	42	Broadcasting	0.34356	14.42952	34.4
216	225	9	Broadcasting	0.19751	1.77759	19.8
225	406	181	Satellite Space	0.047869	8.664289	4.8
406	470	64	Amateur	0.17873	11.43872	17.9
470	512	42	Broadcasting	0.060675	2.54835	6.1
512	608	96	Broadcasting	0.26453	25.39488	26.5
608	698	90	Broadcasting	0.040389	3.63501	4.0
698	806	108	Broadcasting	0.35093	37.90044	35.1
806	902	96	Broadcasting	0.09924	9.52704	9.9
902	928	26	GSM, Land Mobile	0.0072915	0.189579	0.7
928	1000	72	Broadcasting	0.29214	21.03408	29.2
1000	1240	240	Broadcasting, Satellite	0.034669	8.32056	3.5
1240	1300	60	Amateur	0.0061554	0.369324	0.6
1300	1400	100	Satellite	0.0395	3.95	4.0
1400	1525	125	Fixed Mobile	0.15158	18.9475	15.2
1525	1710	185	Satellite	0.37198	68.8163	37.2
1710	1850	140	Fixed Mobile	0.38536	53.9504	38.5
1850	1990	140	Fixed Mobile	0.35121	49.1694	35.1
1990	2110	120	Fixed Mobile	0.0066952	0.803424	0.7
2110	2200	90	Fixed Mobile	0.2319	20.871	23.2
2200	2300	100	Fixed Mobile	0.0091076	0.91076	0.9
2300	2360	60	Fixed Mobile	0.004666	0.27996	0.5
2360	2390	30	Fixed Mobile	0.001883	0.05649	0.2
2390	2500	110	ISM	0.14357	15.7927	14.4
2500	2686	186	Broadcasting	0.14514	26.99604	14.5
2686	2900	214	Astronomy, Radar	0.03031	6.486982	3.0
2900	3000	100	Astronomy, Radar	0.00452	0.4519	0.5
Total		2950		4.2306077	426.75143	
Total Available Spectrum				2950		
Average Spectrum Use (%)				13.60%		

Table 1.1 is from the survey of spectral occupancy in Dublin, Ireland [14]. In the
survey, the available bandwidth is divided into 31 slots, from 30MHz to 3GHz. According to
the data, we first calculate the average spectral occupancy and set 1MHz as one unit of
channel bandwidth and unit increment:

$$\frac{\sum_{i=1}^{31}(bandwidth_i \cdot \%occupancy_i)}{total\ scanned\ bandwidth(MHz)} = 14.47\% \tag{1.28}$$

Therefore, the average 1MHz spectral active probability $P(H_1)$ is 14.47%, and spectral idle probability $P(H_0)$ is 85.53%.

Assume noise spectrum power $N_0 = 10^{-12} Watt/Hz$, received PU power $P_P^h = 10^{-6}$ Watt. Assume BPSK modulation is used for decision reports from each CR to DFC, and we choose to implement a low power consumption XTend transceiver [15] for CR to transmit its signals. This type of transceiver operates at frequency range of 902-928MHz, and it is compatible for DigiMeshTM networking topology. We set 9.6Kb/s as transmission rate in order to minimize reporting channel error probability since it is the minimal transmission rate, and set transmission power from CR to FC (P_d) and transmission power from CR to SR to be 1W, or 30dBm. Therefore, we have bit energy $E_D = 0.104mJ$. Furthermore, according to survey on path loss in German cities shown in Figure 1.4, we choose the pass loss exponent to be 2.7. Assume all the CRs are at 50 meters away from FC, and SR is 200 meters away from CR. we have decision report error probability as:

$$P_e = Q\left(\sqrt{\frac{2 \cdot P_d}{N_0 \cdot W}}\right) = Q\left(\sqrt{\frac{2 \cdot P_d \cdot distance^{-2.7}}{N_0 \cdot W}}\right) \qquad (1.29)$$

$$= Q\left(\sqrt{\frac{2 \cdot 1 \cdot 50^{-2.7}}{10^{-12} \cdot 2600,000}}\right) \approx 4.08 \times 10^{-6}$$

According to Figure 1.5 from [16], which shows the relation between power consumption and sampling rate of analog to digital converter (ADC), and since most of the sensing power is consumed by ADC, we can approximate sensing power consumption using power consumption of ADC. When samples are in N bits and $f_S = 2B$, Sensing power can be therefore described as:

$$P_S = N \times 0.0028 \times \left(\frac{2B}{10^3}\right)^{1.0227} (\mu W) \qquad (1.30)$$
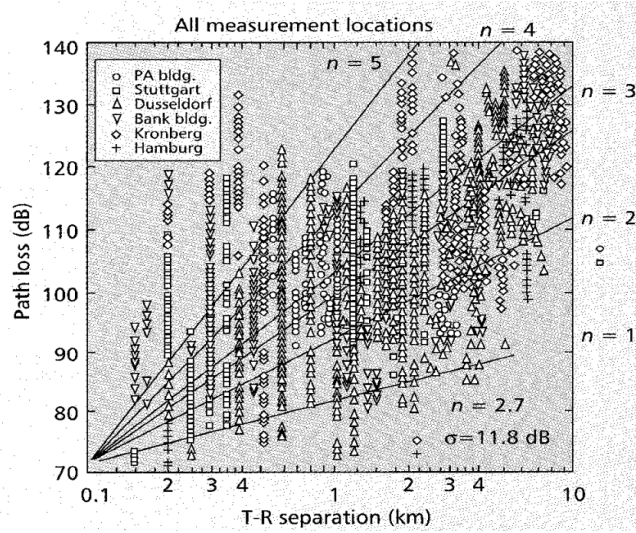
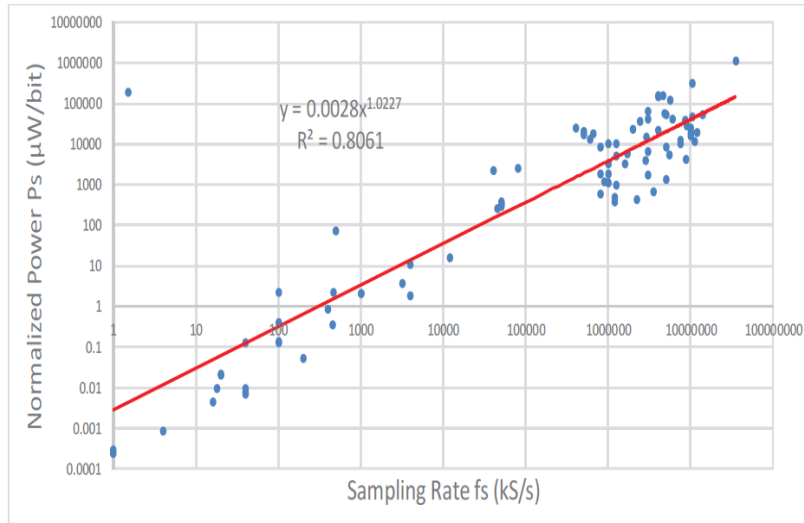*Figure 1.4*: *Path Loss vs Distance in German Cities* [17]

Figure 1.5: *ADC Power Consumption vs Sampling Rate* [16]

Simulation Results Using AND Fusion Rule with Different Misdetection Threshold

All simulations and plots in the thesis use the parameters given below, unless otherwise mentioned in the plots caption.

Table 1.2: Summary of Parameters

Parameters	Parameter Value	Note
$P(H_0)$	85.53%	Average spectral idle probability in Dublin Ireland [11]
$P(H_1)$	14.47%	Average spectral occupied probability in Dublin Ireland [11]
P_e	4.08×10^{-6}	Reporting channel error probability (from CR to DFC)
N_0	1×10^{-12}	Noise power spectral density
Path Loss Exponent	2.7	Found in path loss vs distance in German cities [17]
P_P^h	1×10^{-6} Watt	Received PU signal power measured at CR
P_{CS}	1 Watt	CR transmission power to DFC and SR (30dBm)
P_S	$12 \times 0.0028 \times \left(\frac{2B}{10^3}\right)^{1.0227}$	CR sensing power consumption (ADC power consumption)
T_s	10μS	CR sensing time
T_T	$1 - T_s$	CR transmission time
B_0	100KHz	Bandwidth for each channel when CR senses multiple channels
E_d	104μWatt	Decision report transmission energy consumption
E_C	2.5×10^{-5}	CR circuit energy consumption in one frame cycle
d_{CS}	200 meters	Distance between CR and secondary receiver (SR)
N	12	Number of bits for ADC to scale the samples

As shown above in Figure 1.6 a, X-axis is the number of cooperative sensing CRs, Y-axis is sensing bandwidth in hertz, and Z-axis (illustrated in color) is energy efficiency. There exist maximal energy efficiency in AND fusion rule with a preset misdetection threshold (labeled in each subplot). In the simulation, we found out as we increase misdetection threshold, we can achieve higher maximal energy efficiency. The optimal number of CR decreases and optimal sensing bandwidth increases as misdetection threshold increases.
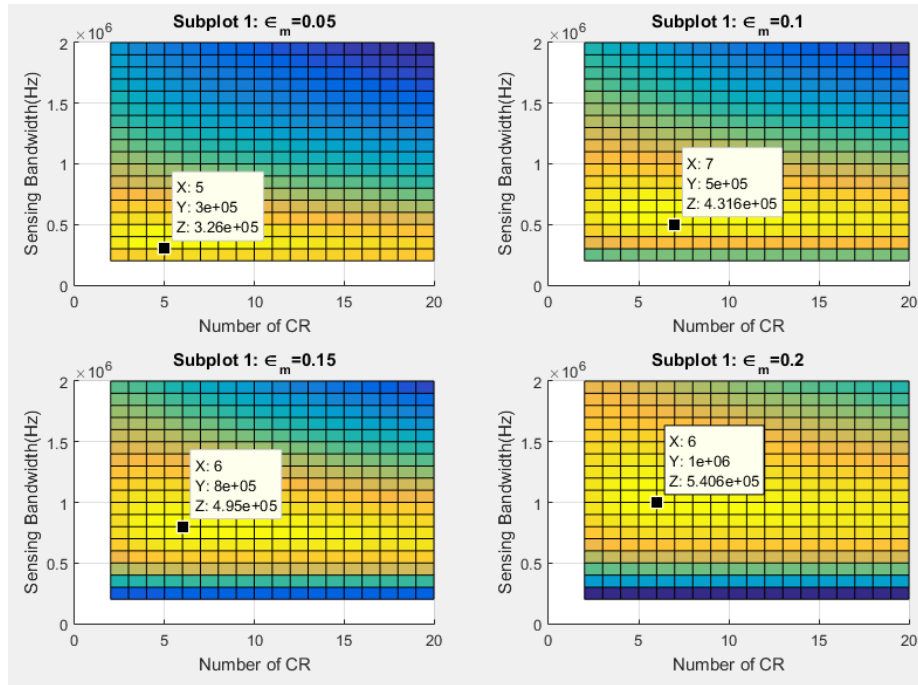
Figure 1.6 a: Energy Efficiency (Z-axis) vs Number of CR (X-axis) vs Sensing Bandwidth (Y-axis) Using AND Fusion Rule with Different $\in_m$
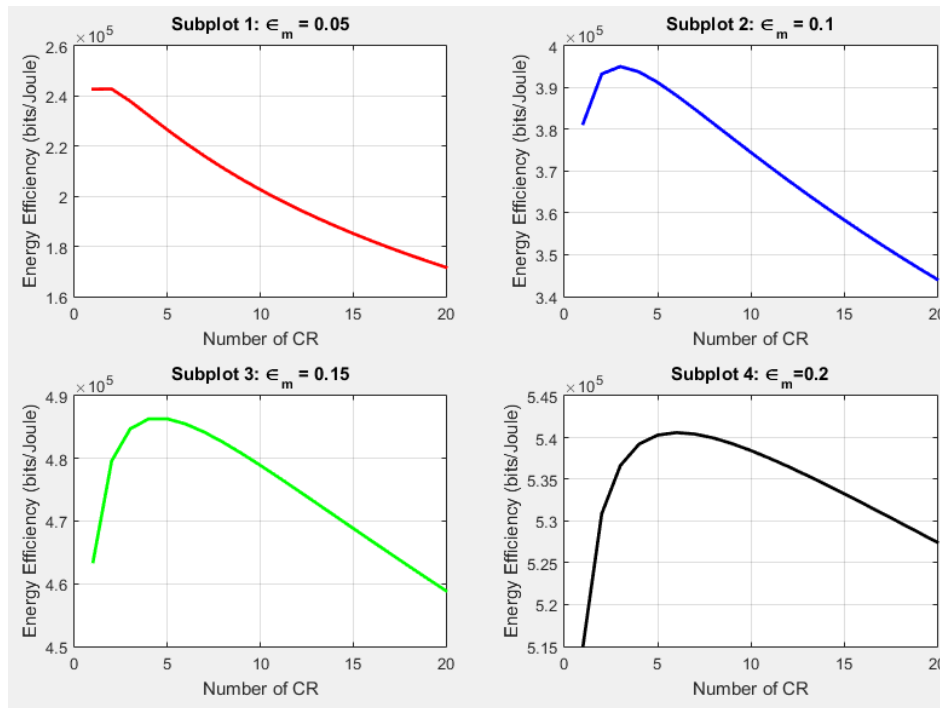

Figure 1.6 b: Energy Efficiency vs Number of CR Using AND Fusion Rule (B=1MHz)

Next, we let the primary user active probability $P(H_1)$ as a variable, and find optimal number of coop CR for given different $P(H_1)$. Finally, we plot the relation between $P(H_1)$ and optimal number of coop CR. We set misdetection threshold to 0.1 and sensing bandwidth to 1MHz.
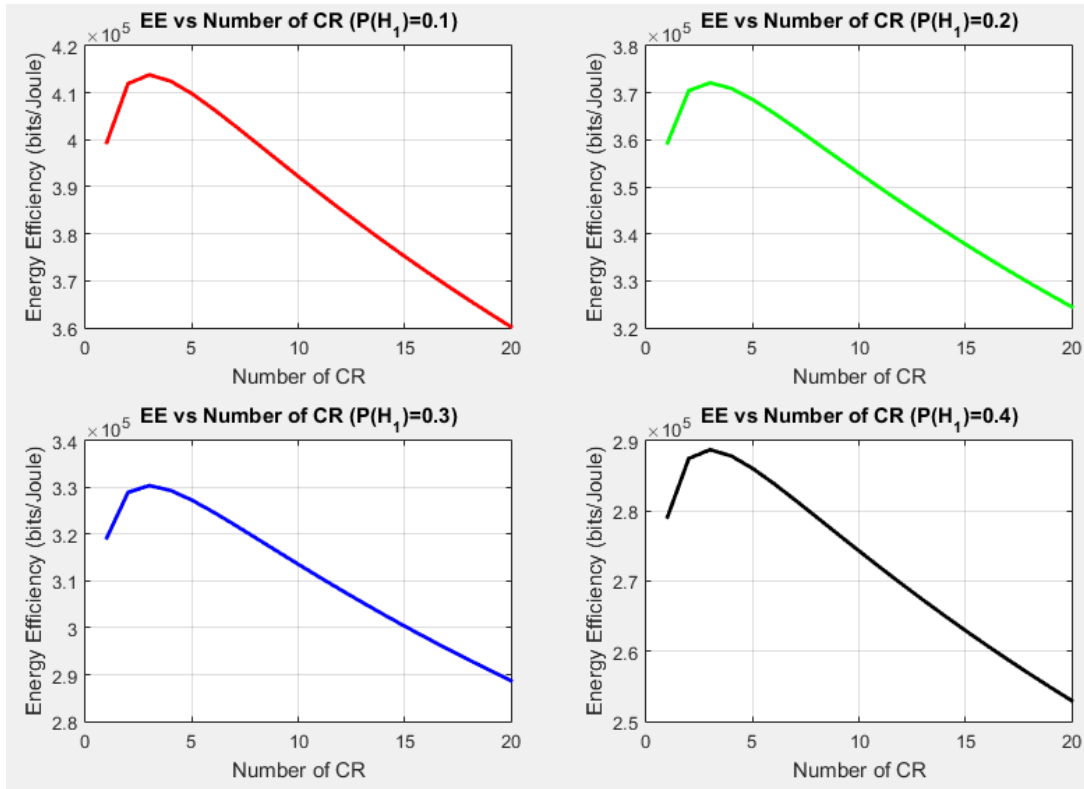

Figure 1.6 c: *Energy Efficiency vs Number of CR with Changing $P(H_1)$ ($\in_m = 0.1$)*

As we can see from above, energy efficiency increases as we decreases $P(H_1)$.

CHAPTER IV

SENSING MULTIPLE CHANNELS OF SAME BANDWIDTH

In this chapter, we study energy efficiency in CR networks when CR senses multiple channels and each channel with same bandwidth. We can visualize this problem by assuming there are several PUs nearby and each of them uses its own channel with no overlap with each other. CR senses multiple channels each with same bandwidth instead of single channel with changing bandwidth. For simplicity, we assume each channel has same bandwidth.

One Cognitive Radio with Two Primary Users

Assuming there are 2 PUs nearby, and each which probability of transmission $P(H_1) = 0.145$ and probability of idle (no transmission) $P(H_0) = 1 - P(H_1) = 0.855$
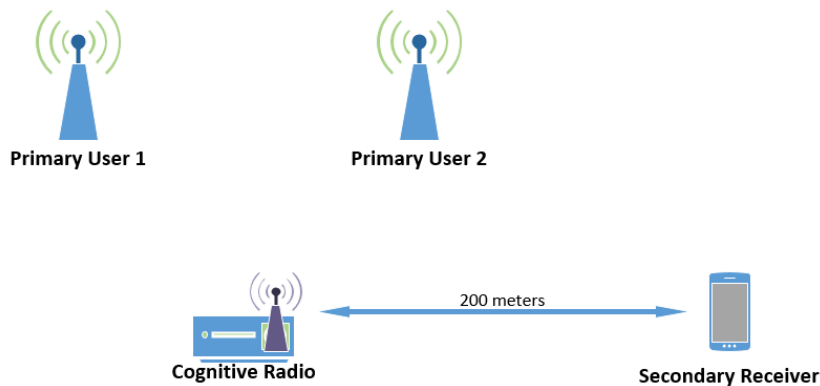

Figure 2.1: CR Network with 1 CR and 2 PUs

The tradeoff between number of PUs and CR transmission bandwidth can be described as below: when CR increases its sensing and transmission bandwidth, the network will have an increase of data transmission rate as described in paper [18]. However, increasing CR bandwidth will inevitably causing collision with PU's transmission. Figure 2.2 illustrated this scenario:

Figure 2.2: CR Transmission Interfere with Primary Transmissions

As shown in Fig.2, assume PU use different channels and as CR transmission bandwidth increases, the transmission bandwidth of CR intersects (overlaps) with primary transmission bandwidth.

Next, we look at misdetection probability. In general, we have misdetection probability described as:

$$P_m = P\left(\widehat{\mathcal{H}_0}\,\middle|\,\mathcal{H}_1\right) = \frac{P(\widehat{\mathcal{H}_0}, \mathcal{H}_1)}{P(\mathcal{H}_1)} \tag{2.1}$$

As shown in equation (2.1), $\mathcal{H}_0$ is when PU is inactive (idle) and $\mathcal{H}_1$ is when PU is active (busy). Then, we extend this representation for 2 PUs present: H_{00} means when PU1 and PU2 are both idle, H_{01} is when PU1 is idle while PU2 is active. And so on for S PUs, in which the representation of their activity is described in form of $H_{01\ldots10}$, as we will discuss in the next section. When there are two PUs, we have misdetection probability:

$$P_{m,2} = P\left(\widehat{\mathcal{H}_0}\,\middle|\,\mathcal{H}_1\right) = \frac{P(\widehat{\mathcal{H}_0}, \mathcal{H}_1)}{P(\mathcal{H}_1)}$$

$$= \frac{P(\widehat{\mathcal{H}_0} \cap \mathcal{H}_1)}{P(\mathcal{H}_1)} = \frac{P(\widehat{\mathcal{H}_0} \cap (H_{01} \cup H_{10} \cup H_{11}))}{P(\mathcal{H}_1)}$$

$$= \frac{P(\widehat{\mathcal{H}_0} \cap H_{01}) + P(\widehat{\mathcal{H}_0} \cap H_{10}) + P(\widehat{\mathcal{H}_0} \cap H_{11}) - P(\widehat{\mathcal{H}_0} \cap H_{01} \cap H_{10})}{P(\mathcal{H}_1)}$$

$$- \frac{P(\widehat{\mathcal{H}_0} \cap H_{01} \cap H_{11}) + P(\widehat{\mathcal{H}_0} \cap H_{10} \cap H_{11}) - P(\widehat{\mathcal{H}_0} \cap H_{01} \cap H_{10} \cap H_{11})}{P(\mathcal{H}_1)} \tag{2.2a}$$

We assume primary users' transmissions are independent from each other's with

disjoint channels and same bandwidth B_0; and since H_{01}, H_{10} and H_{11} are mutually exclusive

events, we have: $P(H_{01} \cap H_{10}) = P(H_{01} \cap H_{11}) = P(H_{10} \cap H_{11}) = 0$, and eventually

$$P_{m,2} = \frac{P\left(\left(\widehat{\mathcal{H}_0} \cap H_{01}\right) + \left(\widehat{\mathcal{H}_0} \cap H_{10}\right) + \left(\widehat{\mathcal{H}_0} \cap H_{11}\right)\right)}{P(\mathcal{H}_1)} \tag{2.2b}$$

Furthermore, we have: $\begin{cases} P(\widehat{\mathcal{H}_0} \cap H_{01}) = P(\widehat{\mathcal{H}_0}|H_{01})P(H_{01}) \\ P(\widehat{\mathcal{H}_0} \cap H_{10}) = P(\widehat{\mathcal{H}_0}|H_{10})P(H_{10}) \\ P(\widehat{\mathcal{H}_0} \cap H_{11}) = P(\widehat{\mathcal{H}_0}|H_{11})P(H_{11}) \end{cases}$ and we adopt following

notations:

$\begin{cases} P(\widehat{\mathcal{H}_0}|H_{01}) = P_{m,01} & \text{\textit{misdetection probability when PU1 is idle and PU2 is active}} \\ P(\widehat{\mathcal{H}_0}|H_{10}) = P_{m,10} & \text{\textit{misdetection probability when PU1 is active and PU2 is idle}} \\ P(\widehat{\mathcal{H}_0}|H_{11}) = P_{m,11} & \text{\textit{misdetection probability when PU1 and PU2 are both active}} \end{cases}$

In which $P_{m,01}$ means PU 1 is idle and PU 2 is active. Then the misdetection

probability when there are 2 PU can be expressed as:

$$P_{m,2} = \frac{P(\widehat{\mathcal{H}_0}|H_{01})P(H_{01}) + P(\widehat{\mathcal{H}_0}|H_{10})P(H_{10}) + P(\widehat{\mathcal{H}_0}|H_{11})P(H_{11})}{P(H_{01}) + P(H_{10}) + P(H_{11})}$$

$$= \frac{P_{m,01}P(H_{01}) + P_{m,10}P(H_{10}) + P_{m,11}P(H_{11})}{P(H_{01}) + P(H_{10}) + P(H_{11})} \tag{2.2c}$$

For misdetection probability, we have from equation (1.7) with bandwidth B_0:

$$P_m = 1 - P_d = 1 - Q_{T_S B_0}\left(\sqrt{2\gamma}, \sqrt{\lambda}\right) \tag{2.3}$$

In which γ is SNR from primary user and received at CR; λ is detection threshold and

T_S is sensing time. In the case when there are 2 PUs, we have different SNR from PU to CR,

depend on how many of the PU is active:

$$\begin{cases} \gamma_{\text{PC01}} = \dfrac{\text{P}_\text{P}^h}{2N_0 B_0} \ (SNR\ from\ PU, received\ by\ CR; when\ PU1\ is\ idle\ and\ PU2\ is\ active) \\[3mm] \gamma_{\text{PC10}} = \dfrac{\text{P}_\text{P}^h}{2N_0 B_0} \ (SNR\ from\ PU, received\ by\ CR; when\ PU1\ is\ idle\ and\ PU2\ is\ active) \\[3mm] \gamma_{\text{PC11}} = \dfrac{2\text{P}_\text{P}^h}{2N_0 B_0} \ (SNR\ from\ PU, received\ by\ CR; when\ both\ PU\ are\ active) \end{cases}$$

In which P_P^h is transmission power from PU and measured at CR. The misdetection

probability described in equation (2.2) can be detailed into:

$$P_{m,2} = \frac{P_{m,01}P(H_{01}) + P_{m,10}P(H_{10}) + P_{m,11}P(H_{11})}{P(H_{01}) + P(H_{10}) + P(H_{11})} = \frac{P(H_0)P(H_1)\big(P_{m,01} + P_{m,10}\big) + P(H_1)^2 P_{m,11}}{1 - P(H_0)^2}$$

$$= \frac{P(H_0)P(H_1)\big[1 - Q_{2T_S B_0}\big(\sqrt{2\gamma_{PC01}}, \sqrt{\lambda}\big) + 1 - Q_{2T_S B_0}\big(\sqrt{2\gamma_{PC10}}, \sqrt{\lambda}\big)\big] + P(H_1)^2 \left(1 - Q_{2T_S B_0}\big(\sqrt{2\gamma_{PC11}}, \sqrt{\lambda}\big)\right)}{1 - P(H_0)^2}$$

$$(\mathbf{2.4})$$

And false alarm probability P_f for 2 PU is:

$$P_{f,2} = \frac{\Gamma\big(2T_s B_0, \frac{\lambda}{2}\big)}{\Gamma\big(2T_s B_0\big)} \tag{$\mathbf{2.5}$}$$

Channel Capacities and Energy Efficiency with Two PUs

Total Channel Capacity can be derived as below:

$Channel\ Capacity$

$$= 2B_0 \log(1 + \gamma_{CS00}) \big(1 - P_{f,2}\big)P(H_0)$$

$$+ 2B_0 \log(1 + \gamma_{CS01}) \, P_{m,01}P(H_0)P(H_1)$$

$$+ 2B_0 \log(1 + \gamma_{CS10}) \, P_{m,10}P(H_1)P(H_0)$$

$$+ 2B_0 \log(1 + \gamma_{CS11}) \, P_{m,11}P(H_1)P(H_1) \tag{$\mathbf{2.6}$}$$

Note that in the first term of equation (2.6), the probability of correct sensing

is $P(H_0)$, since the idle probability for single channel is $P(H_0)$, and the average amount of

channel available given the S channels is $SP(H_0)$. Notations γ_{CSxx} are the signal to noise ratio from CR and received at secondary user (SU), $'x'$ can to replace by digit 0 (PU inactive) or 1 (PU active). And the descriptions for SINR from CR to SR are shown below:

$$\begin{cases} \gamma_{CS00} = \dfrac{P_{CS}^h}{2N_0B_0} \ (SNR \ from \ CR \ to \ SU \ when \ both \ PUs \ are \ idle) \\[2ex] \gamma_{CS01} = \dfrac{P_{CS}^h}{2N_0B_0 + P_P^h} \ (SINR \ from \ CR \ to \ SU, interfered \ by \ the \ first \ PU.) \\[2ex] \gamma_{CS10} = \dfrac{P_{CS}^h}{2N_0B_0 + P_P^h} \ (SINR \ from \ CR \ to \ SU, interfered \ by \ the \ second \ PU.) \\[2ex] \gamma_{CS11} = \dfrac{P_{CS}^h}{2N_0B_0 + 2P_P^h} \ (SINR \ from \ CR \ to \ SU, interfered \ by \ 2 \ PU) \end{cases}$$

The first term on the right hand side of equation (2.6) is the throughput under correct detection given both PUs are idle. The second term is the throughput under misdetection given PU1 is idle and PU2 is active. The third term is the throughput under misdetection given PU1 is active and PU2 is idle. And the last term is the throughput under misdetection given both PUs are active.

Assume frame time (T_F) is 1 second, and sensing time plus transmission time equals frame time: $T_F = T_S + T_T$. The energy efficiency when there are 2 PUs is:

$$Energy \ Efficiency = \frac{transmission \ time \ \times channel \ capacity}{energy \ consumption \ in \ one \ frame}$$

$$= \frac{T_T \left(2B_0 \log(1 + \gamma_{CS00}) \left(1 - P_{f,2}\right)P(H_0) + 2B_0 \log(1 + \gamma_{CS01}) P_{m,01}P(H_0)P(H_1)\right)}{P_s T_S + P_T T_T + P_C(T_S + T_T)}$$

$$+ \frac{T_T\left(2B_0 \log(1 + \gamma_{CS10}) P_{m,10}P(H_1)P(H_0) + 2B_0 \log(1 + \gamma_{CS11}) P_{m,11}P(H_1)^2\right)}{P_s T_S + P_T T_T + P_C(T_S + T_T)} \qquad (2.7)$$

One Cognitive Radio with Multiple PUs

When there are S PUs nearby and all PU uses disjoint channel with same bandwidth B_0.

Then in the case of misdetection, the status of PUs can be from one of them is active to all of

them are active. Which can be described as:

$$\begin{cases} H_{0..01} & PU(1) \ldots PU(S-1) \ are \ idle, only \ PU(S) \ is \ active & --(1 \ PU \ active) \\ H_{0..10} & PU(1) \ldots PU(S-2), PU(r) \ are \ idle, only \ PU(S-1) \ is \ active & --(1 \ PU \ active) \\ & \vdots \\ H_{0..11} & PU(1) \ldots PU(S-2) \ are \ idle, PU(S-1) \ and \ PU(S) \ are \ active & --(2 \ PUs \ active) \\ & \vdots \\ H_{1..11} & PU(1) \ldots PU(S) are \ all \ active & --(all \ PUs \ active) \end{cases}$$

In general, the total amount of 1 PU active cases is $\binom{S}{1} = \frac{S!}{(S-1)!1!} = n$; in which $\binom{a}{b}$

represents b -combinations of a set n. Similarly, the total amount of 2 PUs active cases

is $\binom{S}{2} = \frac{S!}{(S-2)!2!}$. And so on for n PUs are active, which is $\binom{S}{S} = 1$.

Next, we derive misdetection probability when there are S PUs nearby. The general

expression for misdetection probability when there are r PUs is: $P_{m,S} = P\big(\widehat{\mathcal{H}_0}\,|\mathcal{H}_1\big) =$

$\frac{P(\widehat{\mathcal{H}_0},\mathcal{H}_1)}{P(\mathcal{H}_1)}$, and for the numerator*:*

$$P\big(\widehat{\mathcal{H}_0}, \mathcal{H}_1\big) = P\big(\widehat{\mathcal{H}_0}\cap(H_{0\ldots01} \ldots \cup H_{0\ldots11} \ldots \cup H_{1\ldots11})\big)$$

$$= P\big[\big(\widehat{\mathcal{H}_0}\cap H_{0\ldots01}\big) + \cdots + \big(\widehat{\mathcal{H}_0}\cap H_{0\ldots11}\big) + \cdots + \big(\widehat{\mathcal{H}_0}\cap H_{1\ldots11}\big)\big]$$

Since the rest of the term contains mutually exclusive events, we have

$$\big(\widehat{\mathcal{H}_0}\cap H_{0\ldots01}\cap H_{0\ldots11}\big) = \cdots = \big(\widehat{\mathcal{H}_0}\cap H_{0\ldots01} \cap \ldots \cap H_{0\ldots11}\cap H_{1\ldots11}\big) = \emptyset$$

There are $\binom{S}{1} = S$ cases when 1 PU is active, $\binom{S}{2}$ cases when 2 PUs are active and in

general, $\binom{S}{k}$ cases when k PUs are active. We then have for k PUs active:

$$P\left(\widehat{\mathcal{H}_0}\cap H_{\underbrace{0\ldots1}_{k\ 1s}}\right) = \left(P\left(\widehat{\mathcal{H}_0}|H_{00\ldots\underbrace{1\ldots1}_{k\ 1s}}\right) + \cdots + P\left(\widehat{\mathcal{H}_0}|H_{\underbrace{1\ldots1}_{k\ 1s}0\ldots0}\right)\right) \times P(H_0)^{S-k}P(H_1)^k$$

Note that:

$$P\left(\widehat{\mathcal{H}_0}|H_{00\underbrace{..1..1}_{k\,1s}}\right) = \cdots = P\left(\widehat{\mathcal{H}_0}|H_{\underbrace{1..10..0}_{k\,1s}}\right) = 1 - Q_{ST_SB_0}\left(\sqrt{2\frac{kP_P^h}{SN_0B_0}}, \sqrt{\lambda}\right)$$

Therefore, we have:

$$P\left(\widehat{\mathcal{H}_0}\cap H_{\underbrace{0...1}_{k\,1s}}\right)$$

$$= P(H_0)^{S-k}P(H_1)^k\underbrace{\left[\left(1 - Q_{ST_SB_0}\left(\sqrt{2\frac{kP_P^h}{SN_0B_0}}, \sqrt{\lambda}\right)\right) + \cdots + \left(1 - Q_{ST_SB_0}\left(\sqrt{2\frac{kP_P^h}{SN_0B_0}}, \sqrt{\lambda}\right)\right)\right]}_{totally\ \binom{S}{k}\ terms\ and\ each\ term\ is\ identical\ with\ one\ another}$$

$$= \binom{S}{k}P(H_0)^{S-k}P(H_1)^k\left[1 - Q_{ST_SB_0}\left(\sqrt{2\frac{kP_P^h}{SN_0B_0}}, \sqrt{\lambda}\right)\right]$$

Hence, when there are S PUs nearby, the numerator of equation (2.1) is:

$$P(\widehat{\mathcal{H}_0}, \mathcal{H}_1) = \binom{S}{1}P(H_0)^{S-1}P(H_1)\left[1 - Q_{ST_SB_0}\left(\sqrt{2\frac{P_P^h}{SN_0B_0}}, \sqrt{\lambda}\right)\right]$$

$$+ \binom{S}{2}P(H_0)^{S-2}P(H_1)^2\left[1 - Q_{ST_SB_0}\left(\sqrt{2\frac{2P_P^h}{SN_0B_0}}, \sqrt{\lambda}\right)\right]$$

$$+$$

$$\vdots$$

$$+$$

$$+ \binom{S}{S}P(H_0)^{S-S}P(H_1)^S\left[1 - Q_{ST_SB_0}\left(\sqrt{2\frac{P_P^h}{N_0B_0}}, \sqrt{\lambda}\right)\right]$$

$$= \sum_{k=1}^{S}\binom{S}{k}P(H_0)^{S-k}P(H_1)^k\left[1 - Q_{ST_SB_0}\left(\sqrt{2\frac{kP_P^h}{SN_0B_0}}, \sqrt{\lambda}\right)\right]$$

We let: $P_{m,S,k} = 1 - Q_{ST_SB_0}\left(\sqrt{2\frac{kP_P^h}{SN_0B_0}}, \sqrt{\lambda}\right)$, which is the misdetection probability

when k out of S PU is (are) active. And Let:

$$P(\widehat{\mathcal{H}}_0, \mathcal{H}_{1,k}) = \binom{S}{k} P(H_0)^{(S-k)} P(H_1)^k \left[1 - Q_{ST_SB_0}\left(\sqrt{2\frac{kP_P^h}{SN_0B_0}}, \sqrt{\lambda}\right)\right]$$

We then have for S PUs, misdetection probability can be described as:

$$P_{m,S} = \frac{P(\widehat{\mathcal{H}}_0, \mathcal{H}_1)}{P(\mathcal{H}_1)} = \frac{P(\widehat{\mathcal{H}}_0, \mathcal{H}_{1,1}) + P(\widehat{\mathcal{H}}_0, \mathcal{H}_{1,2}) + \cdots + P(\widehat{\mathcal{H}}_0, \mathcal{H}_{1,S})}{1 - P(H_0)^S}$$

$$= \frac{\sum_{k=1}^S \binom{S}{k} P_{m,S,k} P(H_0)^{(S-k)} P(H_1)^k}{1 - P(H_0)^S} \tag{2.8}$$

Similarly to Equation (2.5), false alarm probability for r PUs is:

$$P_{f,S} = P(\widehat{\mathcal{H}}_1 | \mathcal{H}_0) = \frac{\Gamma(ST_sB_0, \frac{\lambda}{2})}{\Gamma(ST_sB_0)} \tag{2.9}$$

Channel Capacities and Energy Efficiency with Multiple PUs

According to Equation (2.6) and Part 4, the total Channel Capacity when there are S PUs is shown below:

$Channel\ Capacity\ (S\ PUs)$

$$= SB_0\left(\log\left(1 + \frac{P_{CS}^h}{SN_0B_0}\right)(1 - P_{f,S})P(H_0)\right.$$

$$+ \binom{S}{1}\log\left(1 + \frac{P_{CS}^h}{SN_0B_0 + P_P^h}\right)P_{m,S,1}P(H_0)^{S-1}P(H_1)$$

$$+ \binom{S}{2}\log\left(1 + \frac{P_{CS}^h}{SN_0B_0 + 2P_P^h}\right)P_{m,S,2}P(H_0)^{S-2}P(H_1)^2 + \cdots$$

$$+ \binom{S}{S}\log\left(1 + \frac{P_{CS}^h}{S(N_0B_0 + P_P^h)}\right)P_{m,S,S}P(H_0)^{S-S}P(H_1)^S\right)$$

$$= SB_0 \left(\log\left(1 + \frac{P_{CS}^h}{SN_0B_0}\right)(1 - P_{f,S})P(H_0) \right.$$

$$\left. + \sum_{k=1}^{S} \binom{S}{k} \log\left(1 + \frac{P_{CS}^h}{SN_0B_0 + kP_P^h}\right) P_{m,S,k}P(H_0)^{S-k}P(H_1)^k \right) \qquad (2.10)$$

In general, when CR transmission time is T_T, energy efficiency of the CR network, when there are S PUs, is:

$$Energy\ Efficiency = \frac{T_T \times channel\ capacity}{power\ consumption\ of\ one\ frame}$$

$$= T_T SB_0 \left(\frac{\log\left(1 + \frac{P_{CS}^h}{SN_0B_0}\right)(1 - P_{f,S})P(H_0) + \sum_{l=1}^{S} \binom{S}{l} \log\left(1 + \frac{P_{CS}^h}{SN_0B_0 + lP_P^h}\right) P_{m,S,l}P(H_0)^{S-l}P(H_1)^l}{P_S T_S + P_T T_T + P_C(T_S + T_T)} \right)$$

$$(2.11)$$

Setting Misdetection Threshold on CRs

In order to protect PU transmission and avoid excessive interference by CR transmission, we need to set a misdetection threshold ϵ_m.

When there are S PUs in the sensing environment, the misdetection threshold should be able to protect each PU, and we assume each are protected with same misdetection threshold. Therefore, we have to make sure that the no matter how many PUs are active, misdetection probability has to always less than or equals to the preset misdetection threshold ϵ_m. Or:

$$\begin{cases} P_{m,S,1} \leq \epsilon_m \\ P_{m,S,2} \leq \epsilon_m \\ \quad\vdots \\ P_{m,S,S} \leq \epsilon_m \end{cases}$$

Next, we look at the misdetection for S PUs when k of them are active $P_{m,S,k}$:

$$P_{m,S,k} = 1 - Q_{ST_S B_0}\left(\sqrt{2\frac{kP_P^h}{SN_0 B_0}}, \sqrt{\lambda}\right) \tag{2.12}$$

As we can see from above, when the number of active PU (k) increases, term

$\sqrt{2\frac{kP_P^h}{SN_0 B_0}}$ increases and $Q_{ST_S B_0}\left(\sqrt{2\frac{kP_P^h}{SN_0 B_0}}, \sqrt{\lambda}\right)$ increases, which leads to decrease in

misdetection probability. Therefore; misdetection probability decreases as the amount of

active PU increase: $P_{m,S,k} \geq P_{m,S,k+1}$. Intuitively thinking, as more PU are active, CR will

receive a stronger signal which leads to a decrease in sensing error probability. Moreover, we

can see that for given S PUs, we can assure misdetection probability is below preset

misdetection threshold ϵ_m if the misdetection probability of 1 PU active is below ϵ_m:

$$if: \quad P_{m,S,1} = 1 - Q_{ST_S B_0}\left(\sqrt{2\frac{P_P^h}{SN_0 B_0}}, \sqrt{\lambda}\right) \leq \epsilon_m$$

$$then: \quad P_{m,S,k} = 1 - Q_{ST_S B_0}\left(\sqrt{2\frac{kP_P^h}{SN_0 B_0}}, \sqrt{\lambda}\right) \leq \epsilon_m \quad for \ k \in 1,2,\ldots,S$$

Next, we look at the simulation of ϵ_m, which is the misdetection probability of one

PU active while the rest are idle:

As we can see from Figure 2.3 below, for given amount of PUs, misdetection

probability increases as preset detection threshold of each sample $\frac{\lambda}{2T_s B}$ increases (we use $\frac{\lambda}{2T_s B}$

since it is more understandable, for example, $\frac{\lambda}{2T_s B} = 1.5$ means we set detection threshold to

be 1.5 times noise power). Intuitively thinking, the decrease of detection threshold is
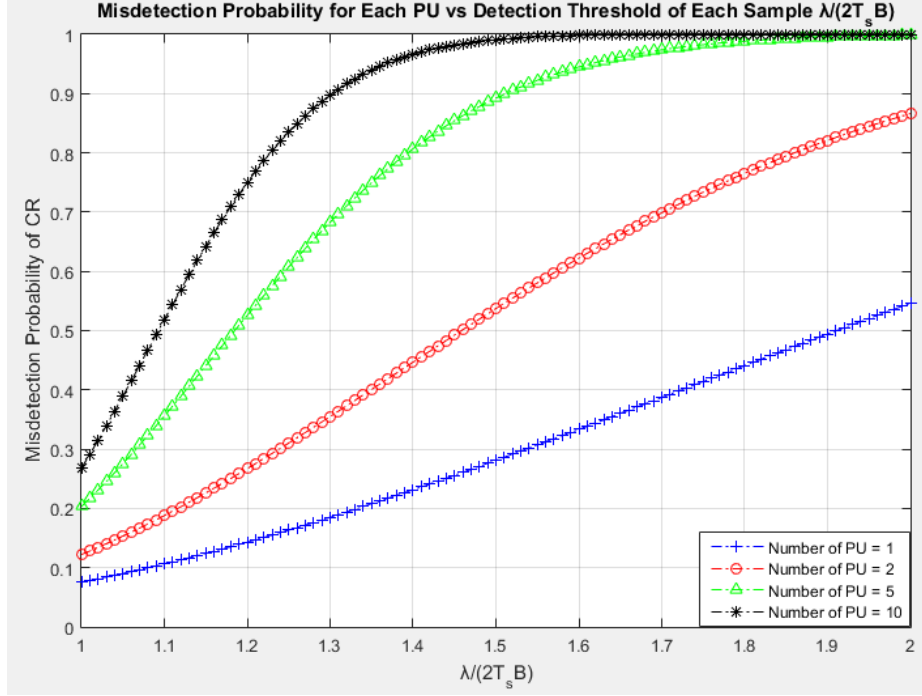
equivalent to increase CR sensing sensitivity.

Figure 2.3: Misdetection Probability vs Detection Threshold of Each Sample

Next, we derive the relation between misdetection and detection threshold of each

sample. Following Chapter II section 4 and considering channel error probability, we have:

$$\lambda \leq ncx2inv\left(\epsilon_m, ST_SB, ST_SB\frac{P_P^h}{SN_0B_0}\right) \tag{2.13}$$

Note that K is the number of CR and k is the number of active PU.

According to equation (2.9) and (2.11), for a given false alarm probability, in order to

increase channel capacity and energy efficiency, we need to increase misdetection

probability. Furthermore, for a fixed misdetection probability, in order to increase channel

capacity and energy efficiency, we should decrease false alarm probability. From equation

(2.9), we can easily see that in order to decrease false alarm probability, we should

maximize λ. Therefore, λ should be maximized in order to maximize energy efficiency (in

the meanwhile, misdetection probability, as a function of λ, should also equals to or less than

preset misdetection threshold ϵ_m).

In general; for a given number of PUs, maximal energy efficiency is achieved when

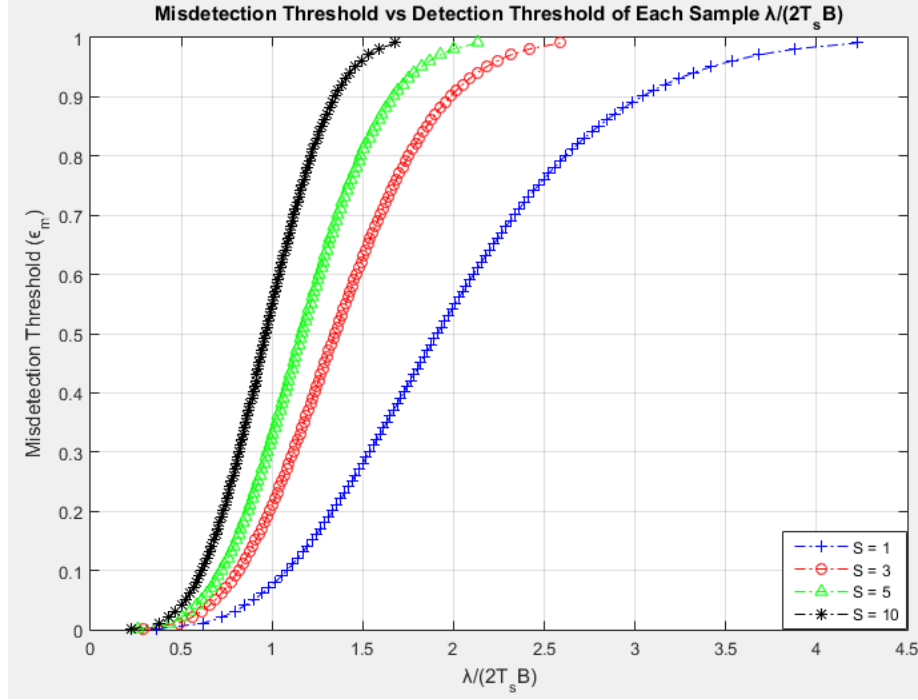$$\lambda_{opt} = ncx2inv\left(\epsilon_m, ST_SB, ST_SB\frac{P_P^h}{SN_0B_0}\right) \tag{2.14}$$

Figure 2.4: Misdetection Threshold vs Detection Threshold of Each Sample

Next, we generate and plot maximal energy efficiency for given misdetection threshold with different number of sensing channels (each channel with 100 KHz bandwidth). As shown in Fig.8 below, we calculated maximal energy efficiency given numbers of PUs under the constraint of preset misdetection threshold, in order to protect PU.
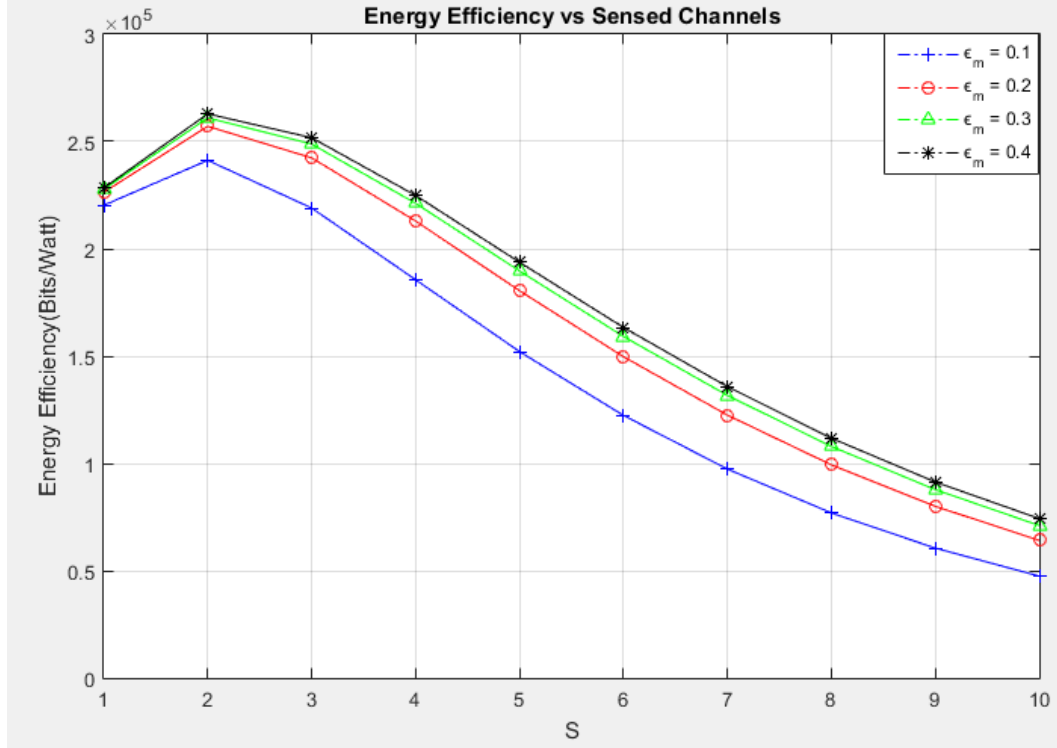
Figure. 2.5: *Maximal Energy Efficiency vs Sensing Bandwidth (single CR)*

Figure 2.5 above shows achievable Maximal Energy Efficiency for different amount of PUs. Furthermore, the figure shows that amount 0 to 10 PUs, global maximal energy efficiency point is at when there are 4 PUs, with highest misdetection threshold (in this case, $\epsilon_m = 0.4$)

'OR' Fusion Rule with K CRs and S PUs

When there are K CRs in the network, assume they are all identical and have same misdetection and false alarm probability given in the same sensing environment, for 'OR' rule we have:

$$Q_{f,S} = 1 - \left(1 - P_{f,S}^e\right)^K$$
$$Q_{m,S,k} = \left(P_{m,S,k}^e\right)^K \leq \epsilon_m$$

(2.15)

In which $P_{f,S}^{\mathrm{e}}$ is false alarm probability and $P_{m,S,k}^{\mathrm{e}}$ is misdetection probability when k out of S PU are active, and superscript e means we are considering reporting channel error probability. In which:

$$\begin{cases} P_{f,S}^{\mathrm{e}} = P_{f,S}(1 - P_e) + P_e(1 - P_{f,S}) \\ P_{m,S,k}^{\mathrm{e}} = P_{m,S,k}(1 - P_e) + P_e(1 - P_{m,S,k}) \end{cases} \tag{2.16}$$

Correspondently, $Q_{f,S}$ is network false alarm probability; $Q_{m,S,k}$ is misdetection probability at DFC when k out of S PU are active. ϵ_{m} is misdetection threshold. Next, we have the expression for energy efficiency of 1 PU under cooperative sensing environment:

And the energy efficiency for such network is:

$$Energy\ Efficiency = \frac{T_T \times channel\ capacity}{power\ consumption\ of\ one\ frame}$$

$$= T_T S B_0 \left(\frac{\log\left(1 + \frac{P_{CS}^h}{SN_0 B_0}\right)\left(1 - Q_{f,S}\right)P(H_0) + \sum_{l=1}^{S} \binom{S}{l} \log\left(1 + \frac{P_{CS}^h}{SN_0 B_0 + lP_P^h}\right) Q_{m,S,l} P(H_0)^{S-l} P(H_1)^l}{P_T T_T + (P_s \mathrm{T_S} + P_C(\mathrm{T_S} + T_T)) \times K} \right)$$

$$\tag{2.17}$$

Note that sensing power in this case is calculated using equation (2.18) below:

$$P_S = N \times 0.0028 \times \left(\frac{2SB_0}{10^3}\right)^{1.0227} (\mu W) \tag{2.18}$$

Case Study: 'AND' Fusion Rule with k CRs and S PUs

Similar as 'OR' fusion rule, for 'AND' fusion rule with K CRs and S PUs, we have:

$$\begin{cases} Q_{f,S} = (P_{f,S}^{\mathrm{e}})^K \\ Q_{m,S,k} = 1 - (1 - P_{m,S,k}^{\mathrm{e}})^K \leq \epsilon_{\mathrm{m}} \end{cases} \tag{2.19}$$

And the energy efficiency expression for AND fusion rule is same as the energy efficiency for OR fusion rule.

Note that when we are using cooperative sensing, we only need to ensure that the network misdetection probability to be equal or less than misdetection threshold: $Q_m \leq \epsilon_m$, since the final decision is given by DFC.

Refer to section 5 from this chapter, we have:

$$if: \quad P_{m,S,1} = 1 - Q_{ST_S B_0}\left(\sqrt{2\frac{P_P^h}{SN_0B_0}}, \sqrt{\lambda}\right) \leq \epsilon_m$$

$$then: \quad P_{m,S,k} = 1 - Q_{ST_S B_0}\left(\sqrt{2\frac{kP_P^h}{SN_0B_0}}, \sqrt{\lambda}\right) \leq \epsilon_m \quad for \ k \in 1,2,\ldots,S$$

Similarly for cooperative sensing case, since $Q_{m,S,k}$ monotone increases as $P_{m,S,k}^{\mathrm{e}}$ increases, we have:

$$if: \quad Q_{m,S,1} = 1 - \left(1 - P_{m,S,1}^{\mathrm{e}}\right)^K \leq \epsilon_m$$

$$then: \quad Q_{m,S,k} = 1 - \left(1 - P_{m,S,k}^{\mathrm{e}}\right)^K \leq \epsilon_m \quad for \ k \in 2,\ldots,S$$

By using misdetection threshold on each CR instead of for the network, we have:

$$P_{m,S,1} = \frac{P_{m,S,1}^e - P_e}{1 - 2P_e} \leq \frac{1 - (1 - \epsilon_m)^{\left(\frac{1}{K}\right)} - P_e}{1 - 2P_e} \tag{2.20}$$

In which the rightmost term is misdetection threshold at each CR.

And as we described above, by maximizing detection threshold λ, we can maximize misdetection probability while minimize false alarm probability. In cooperative sensing case, we have:

$$\lambda_{opt} = ncx2inv\left(\frac{1 - (1 - \epsilon_m)^{\left(\frac{1}{K}\right)} - P_e}{1 - 2P_e}, ST_S B, ST_S B \frac{P_P^h}{SN_0B_0}\right) \tag{2.21}$$

After finding the optimal detection threshold, we can calculate network false alarm and misdetection probability using equation (2.9), (2.12), (2.16), and (2.19); and use equation (2.17) to calculate energy efficiency of the CR network.

By using the same parameters presented in chapter II, we can plot energy efficiency

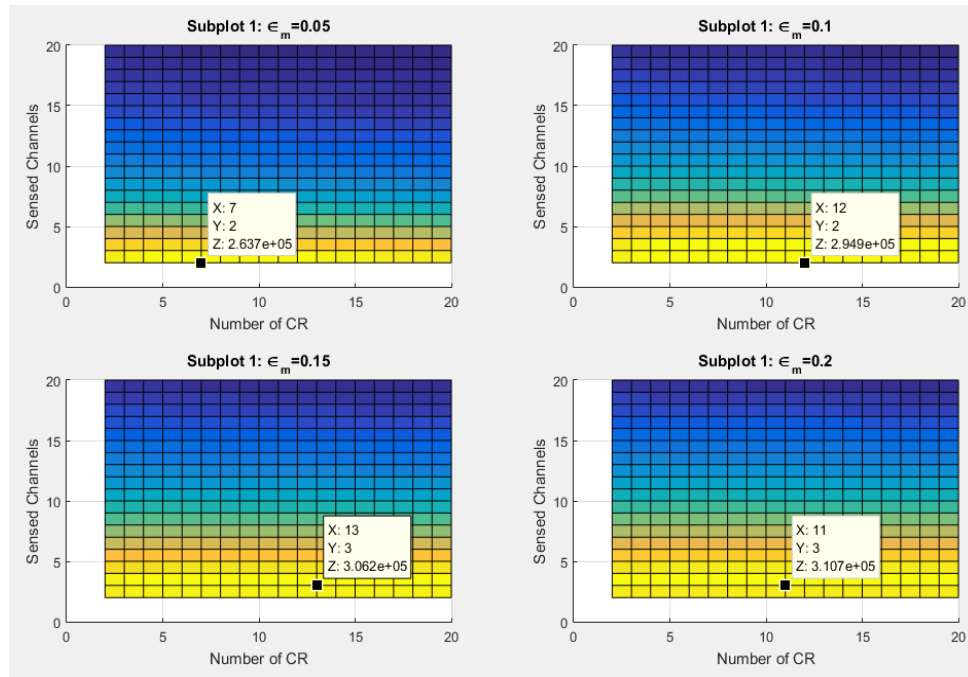vs number of sensing channels vs number of coop CR in Figure 2.6 *a* and *b*:

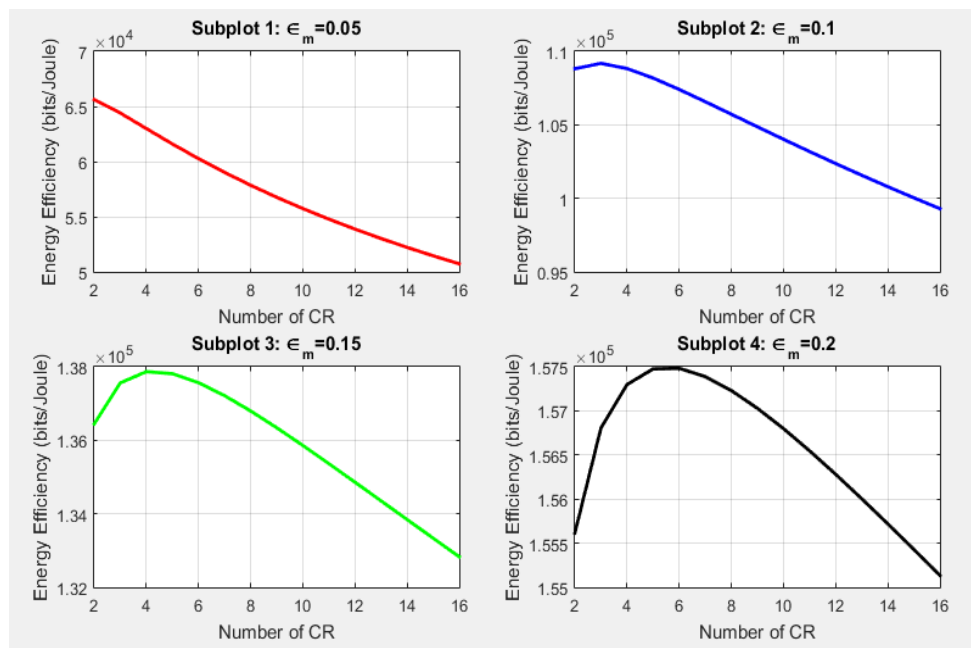***Figure 2.6a:*** *Energy Efficiency vs Number of Sensing Channels vs Number of CRs*

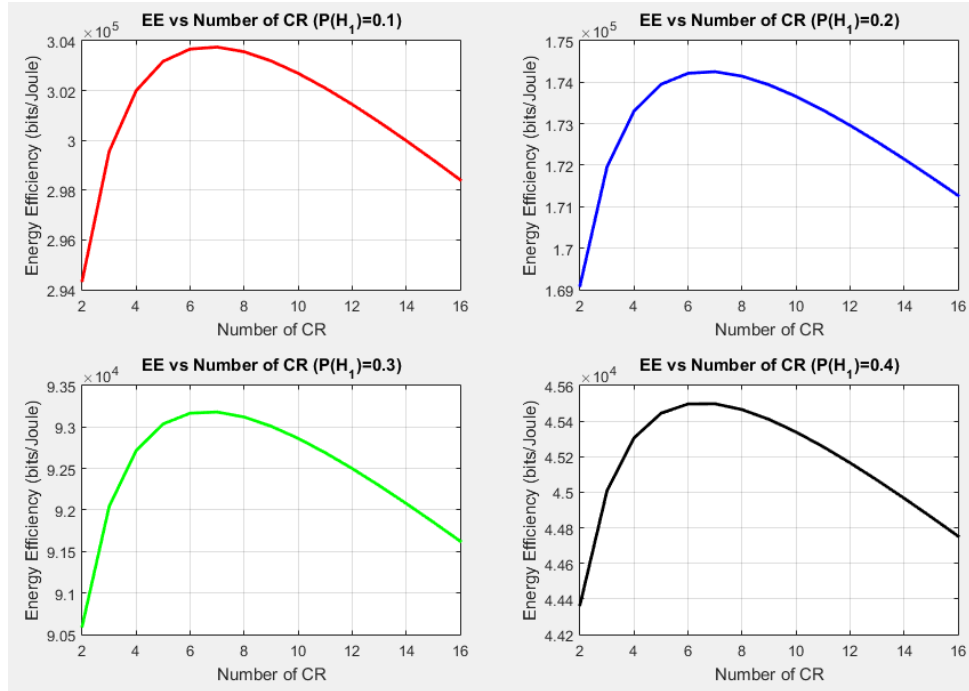***Figure 2.6b:*** *Energy Efficiency vs Number of CRs (AND Fusion Rule, 10 channels)*

Figure 2.6 c: Energy Efficiency vs Number of CR with Changing $P(H_1)$ ($\in_m = 0.1, S = 5$)

Note that sensing bandwidth equals to number of channels times 100 KHz, in which each channel is 100 KHz. By comparing figure 2.6 a and b above with figure 1.5 a and b from chapter 2, we can see that channel sensing method does affects overall energy efficiency of the network. The comparison shows that sensing single channel with changing bandwidth can achieve higher energy efficiency than sensing multiple channels that each with same bandwidth. Moreover, as we increase misdetection threshold, both sensing method shows there is an increase in energy efficiency.

In Chapter II and III, we studied different fusion rules, different sensing methods and focused more on 'AND' fusion rule for its ability to maximize misdetection probability while minimizing false alarm probability for a given amount of CR. Also, we studied on setting misdetection threshold in order to protect PU from CR transmission interference. The case study shows that we can achieve maximal energy efficiency while ensuring the protection of PU in a given environment.

CHAPTER V

CONCLUSION AND FUTURE RESEARCH DIRECTIONS

CR network can uses radio spectrum more efficiently because of its ability to search for idle spectrum and access it opportunistically in order to transmit its own data The study in this thesis was to explore the methods to maximize energy efficiency of the CR network in order to enable green wireless network; Therefore, we can make CR network both spectrum efficient and energy efficient.

In Chapter II, we proved that for a given number of CR, AND fusion rule can maximize energy efficiency. And in Chapter II and III, two different sensing schemes are presented, the first one is to sensing single channel which the bandwidth can be varied, and the next sensing scheme is to sense multiple channels and assume each channel has same bandwidth. Furthermore, we studied about setting misdetection threshold on each CR in order to protect primary user from excessive interference by CR. The case study results show that by presetting misdetection probability and sensing bandwidth, there exist a maximal energy efficiency point. And sensing single channel with varying bandwidth can be more energy efficient than sensing multiple channel slots each with same bandwidth.

Future study on this topic can be to find the optimal sensing time, transmission time or CR transmission power. In addition, more spectrum survey can be conducted in other cities all around the world.

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